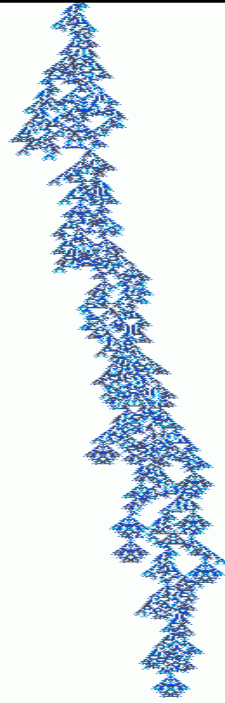




Cellular Automata

Franco Zambonelli
February 2005



Outline

- Part 1: What are CA?
 - Definitions
 - Taxonomy
 - Relevant Examples
- Part 2: CA as Complex Systems
 - Classes of Behavior
 - "The edges of chaos"
- Part 3: Non traditional CA models
 - Asynchronous CA
 - Perturbed CA
- Part 4: Applications of CA
 - Simulation
 - Implications for modern distributed systems



Part 1

- What are CA?



Why CA?

- CA are the simplest complex dynamical systems
 - Simple: easy to define and understand
 - Complex: exhibits very complex behaviors
- Very useful to
 - Understand complex systems
 - Grasp the power of interactions
 - Visualize complex behaviors
 - Simulate complex spatial systems

What are CA: Static Characteristics

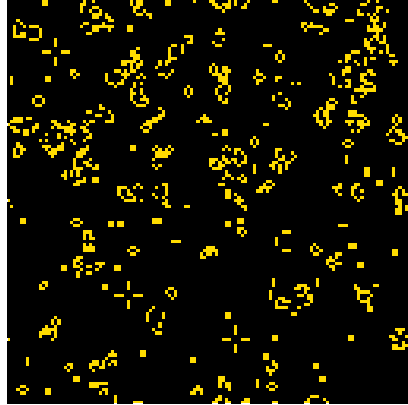
- Discrete Dynamical System
 - $CA = (S, d, N, f)$
- Finite number of cells
 - Interacting in a regular lattice (grid-like, hexagonal, etc.)
- d = dimensional organization of the lattice
 - 1-D, 2-D, 3-D, etc...
- S = Local state of cells
 - Each cell has a local state
 - The state of all cells determines the global state
- f = State Transitions
 - Starting from an initial state S_i
 - Cells change their state based on their current state and of the states of neighbor cells
- N = Neighborhood
 - Which neighbor cells are taken into account in local state transitions

What are CA: Dynamic Characteristics

- Dynamic Evolution
 - Starting from an initial local state
 - Cells change their state based on their current state and of the states of neighbor cells
- Local vs. Global State
 - $S_{\text{global}} = (S_0, S_1, S_2, \dots, S_N)$
 - The Global state evolves as the local states evolves...
- Dynamics of state transitions
 - Synchronous
 - All cells change their state at the same time
 - For each cell i $S_i(t+1) = f(S_i(t), S_k(y) \dots S_n(t))$
 - That is, the global state evolves in a sequence of discrete time steps
 - Asynchronous
 - Cells change their state independently
 - Either by scanning all cells
 - Or by having each cell autonomously trigger its own state transitions
 - Stochastic
 - For both Synchronous and Asynchronous CA, one can consider probabilistic rules for state transitions

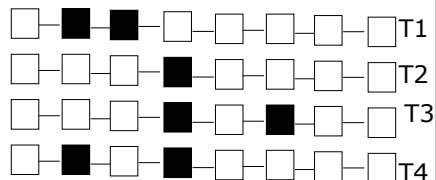
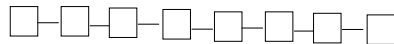
Visualizing CA Evolution

- The success of CA is due to the fact that
 - They are simple complex systems
 - They and their dynamic behavior can be visualized in a very effective manner
- Visualization
 - Associate to each local state a "color" (e.g., for binary states, "black" and "white")
 - Draw the CA grid and see how color changes as the CA evolves...

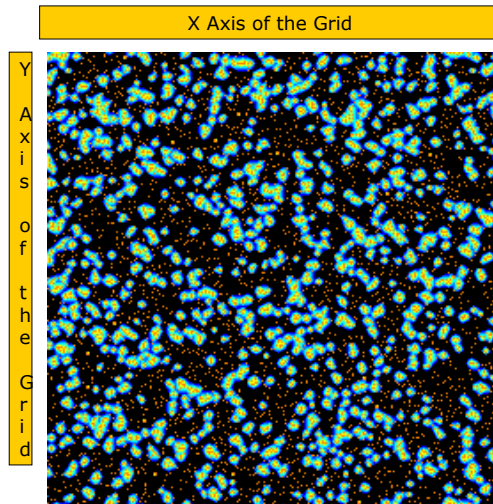


Structure of the Lattice: 1-D CA

- Cells are placed on a straight line
 - connected each other as in a chain
- The evolution of a CA visually represent the evolution in time
 - How the configuration of the global state evolves step after step
 - In a single graph with time descending



Example of 1-D CA Evolution



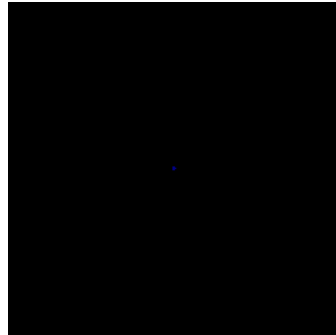
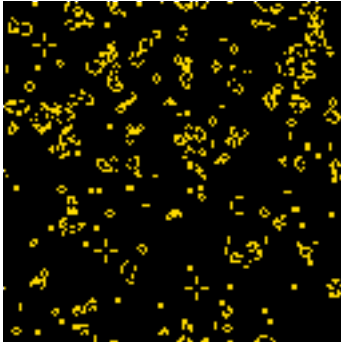
The Time dimension is not explicitly visible

Local States of Cells

- Binary state – simple but very used
 - Each cell has only two states
 - 0 – 1
 - “dead” or “alive”
 - “black” or “white”
- Finite state sets
 - Three, four, five, etc. states.
- Continuous states
 - The state varies over a continuous domain (i.e., the real axis)
 - Often represented as color shades
- Structured states
 - The state can be a “tuple” of values
- The choice depends of what one has to analyze/simulate

Binary vs. Continuous State

- Examples for two different 2- DCAs

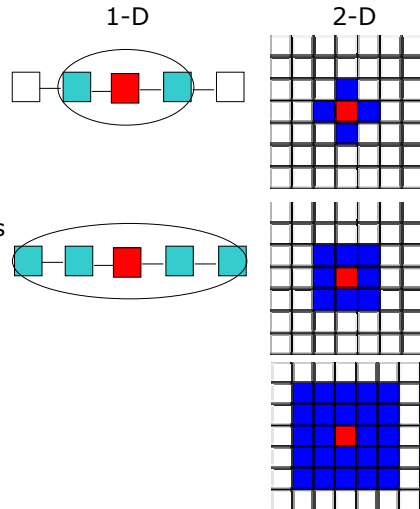


State Transitions

- In general, a cell compute its new state based on
 - Its own current state
 - The current state of a limited number of cells in the neighborhood
- Examples
- For a binary CA
 - $f = \{a \text{ cell in state } S=0 \text{ move to state } S=1 \text{ iff it has 2 neighbors in state } S=1; \text{ a cell in state } S=1 \text{ stays in that state iff it has 1 or 2 neighbors in state } S=1\}$
- For a nearly continuous state CA ($S=0-255$)
 - $f = \{each \text{ cell evaluates its next state } NS \text{ by comparing its current state } CS \text{ and the average value } A \text{ of neighboring cells: if } |CS-A| < 64 \text{ then } NS = A, \text{ if } |CS-A| > 64 \text{ and } A \geq 128 \text{ } NS=255, \text{ if } |CS-A| > 64 \text{ and } A < 128 \text{ } NS=0\}$
- The state transition function clearly has a dramatic impact on the global evolution of the system

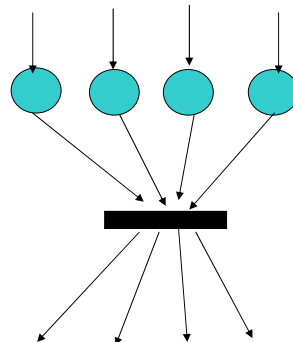
The Neighborhood structure

- How many neighbors are taken into account in state transition?
- Direct neighbors (dist = 1)
 - Only direct neighbors are considered
- Extended neighborhood
 - Neighbors at specific distances are considered
- Weighted neighborhood
 - The "impact" of a neighbor in state transitions decreased with distance (as in physical laws..)
- Non isotropic neighbor structures could also be considered
- All depends on what one has to study/simulate..



Synchronous Dynamics

- Cells change their state as in a synchronization barrier
 - All cells evaluate the transition rules with the local states at time T
 - All of them decide what their local state at time $T+1$ will have to be
 - All together pass to the next state together
 - Over and over for $T+2$, etc.
- SEE THE APPLET ON THE WEB!
- These are the mostly studied one, but do not always reflect the behavior of real-world systems



Asynchronous Dynamics

- In general
 - Each cell evaluate its current state and the current states of neighbor
 - And it performs its own state transition immediately
- However
 - There could be a global controller that decide which cells (and in which order) should do the evaluation (cfr. Object-based systems)
 - Or there could be no global controller, and each cell can autonomously trigger its own internal activities (Cfr. Agent-based systems)
- The behavior of Synch and Asynch CA can be very very different
- Asynch CA are more useful to study decentralized processes and to simulate decentralized systems

CA and Complex Systems (1)

- CA have all the ingredients that can lead to complexity
- It is a system of interconnected components
 - Even if very simply components
 - With very regular interaction topologies
- Local interactions
 - That influences the global behavior
 - Transitive effects → from local to global
- Feedbacks
 - The state of a cell influences its neighbors
 - The state of a cell is influenced by its neighbors
- Non linearity
 - NOT $S_i(t+1) = aS_h(t) + bS_k(t)$
 - BUT $S_i(t+1) = f_{nonlinear}(S_h(t), S_k(t), \dots, S_j(t))$
- Openness and environmental dynamic?
 - Not necessary, but can be introduced to further increase the mess...

CA and Complex Systems (2)

- And CA indeed may exhibit very complex behavior
 - Self-organization (the edge of chaos)
 - Chaotic behavior
- Demonstrating “The Power of Interaction”
 - Very simple components and structure
 - Complexity emerges from interaction dynamics!



The CA “Hall of Fame”

- The Conway’s “Game of Life”
- The family of 256 binary 1-D CA rules
- Totalistic CA



The “Game of Life”

- Discovered by John Conway, mid '70s
- 2-D, synchronous, binary CA
 - Cells are “dead” or “alive”
 - Seems like an ecosystems..
- Neighborhood
 - The Moore neighborhood
- Transition rule
 - *Birth*: a dead cell get alive if it has 3 alive neighbors
 - *Surviving*: an alive cell survive it is has 2 or 3 alive neighbors
 - *Death by overcrowding*: an alive cell dies if it has more than 3 neighbors alive
 - *Death by loneliness*: an alive cell dies if there are less than 2 neighbors alive

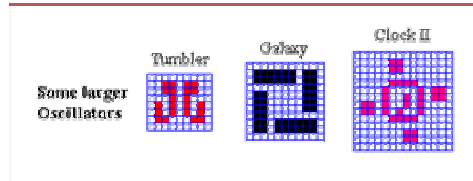
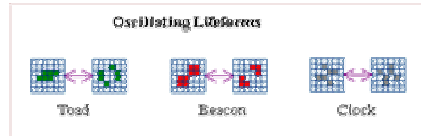


Game of Life: Why it is Interesting?

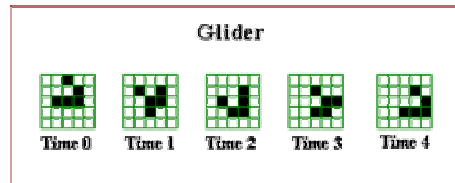
- The Game of Life shows very complex behaviors
 - In general, the evolution shows very complex patterns BUT
- There are stable structures
 - Stable ensembles of close cells that reach a equilibrium
- There are dynamic structures (“gliders”)
 - Ensembles of cells that survive by continuously changing their shape
 - Ensembles of cells that roam in the world
- And there are complex interactions among these structures
 - Some structures annihilates others
 - Some structures generates others

Game of Life: Life-forms Examples

- Oscillators

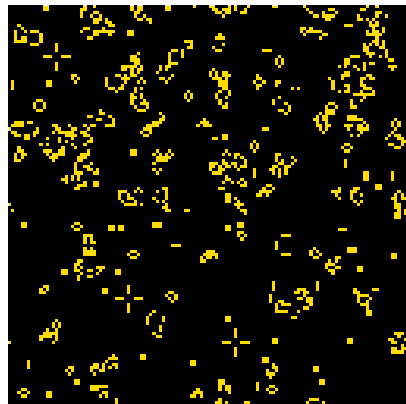
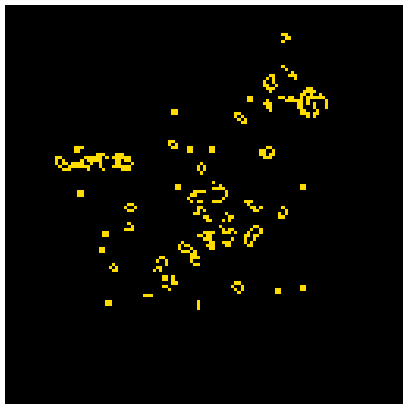


- Gliders



Game of Life: Some Snapshot

- See the world as it evolves...



The 256 Elementary Binary Rules

- 1 Binary, direct neighborhood
 - There are 256 possible rules
 - We can number rules in term of the binary number corresponding to the "next states list"

000	001	010	011	100	101	110	111
0	1	0	1	1	0	1	0

- The above is rule "90" = 01011010



- We can then see how different rules evolve starting from
 - A random initial state
 - A single "seed"

Example: Rule 30

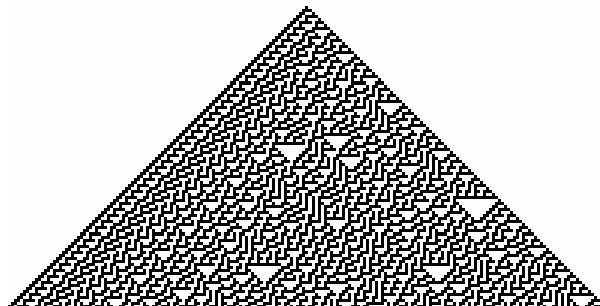
- Rules



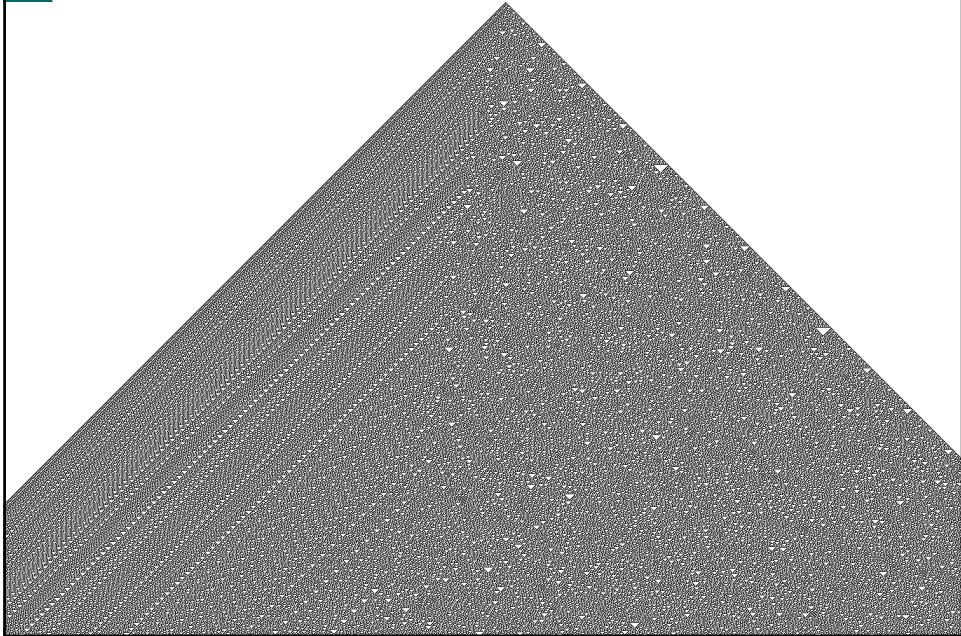
- Steps



- Evolution



More an Rule 30 Evolution...



Example: Rule 110

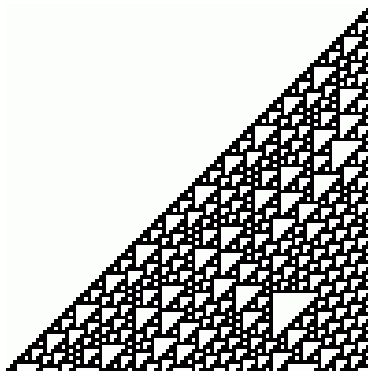
○ Rules



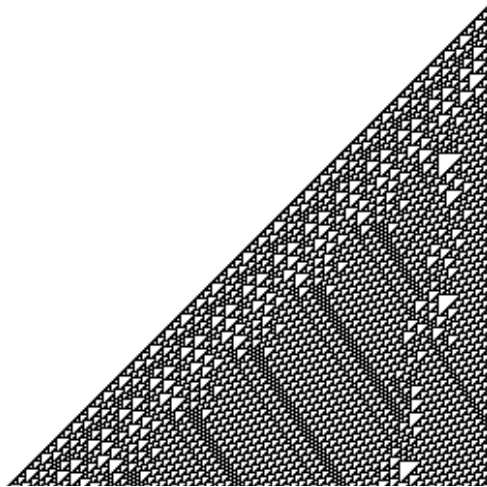
○ Steps



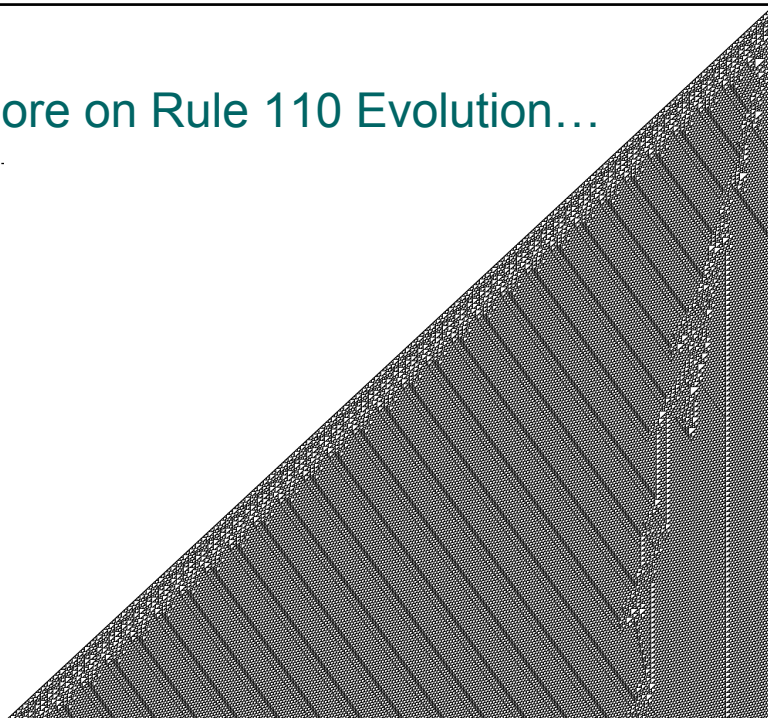
○ Evolution



More on Rule 110 Evolution...

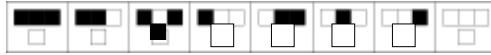


More on Rule 110 Evolution...



Example: Rule 32

- Rules



- Steps & Evolution:

- It is clearly that all cells rapidly becomes white

Example: Rule 250

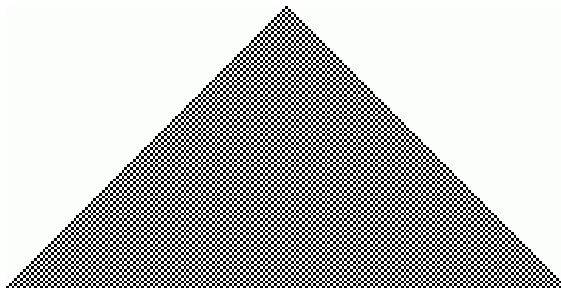
- Rules



- Steps



- Evolution



Example: Rule 90

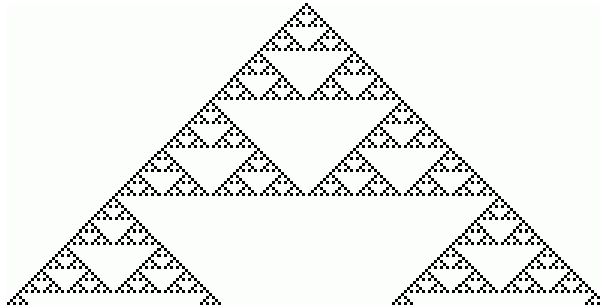
- Rules



- Steps



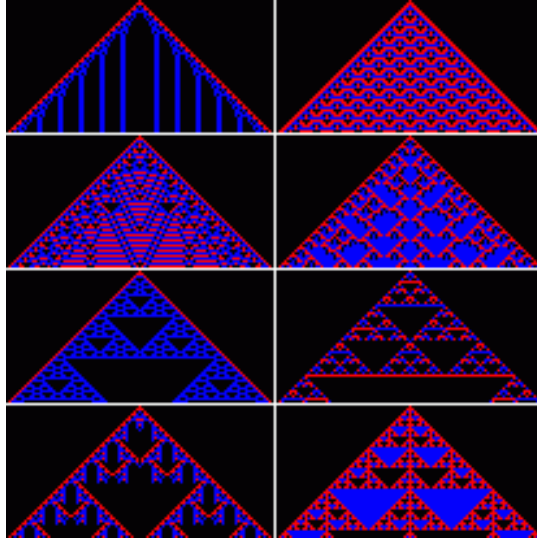
- Evolution



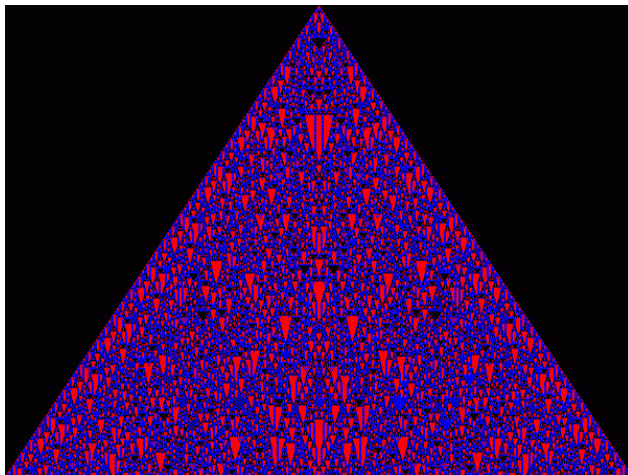
Totalistic CA

- In the rules, the state transitions in cells consider
 - The average of all neighborhood
 - Rather than the specific configurations of cells
- Let us now consider, for example
 - 1 Totalistic CA, 3- states (3 colors) with 3 neighbors

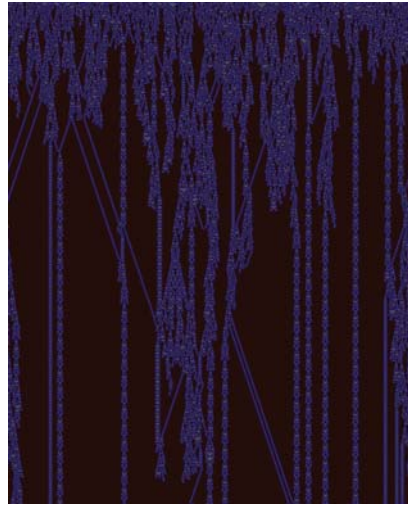
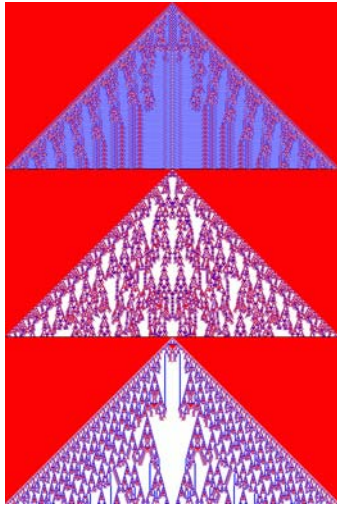
Examples of Totalistic Patterns (1)



Examples of Totalistic Patterns (2)



Examples of Totalistic Patterns (3)



Part 2

- CA as Complex Systems



CA as Complex Systems

- We have seen different types of CA
 - Still, we can easily recognize that, for all types of CA, and depending on rules, the dynamic evolution of CA can fall under a limited number of classes
- The fact is:
- The possible behaviors of CA fall into a limited number of classes
 - Which are “universal” for CA
 - That is, apply to any type of CA
 - Which are “paradigmatic” of any type of complex system
 - That is, are representative of the behavior of all types of systems in nature

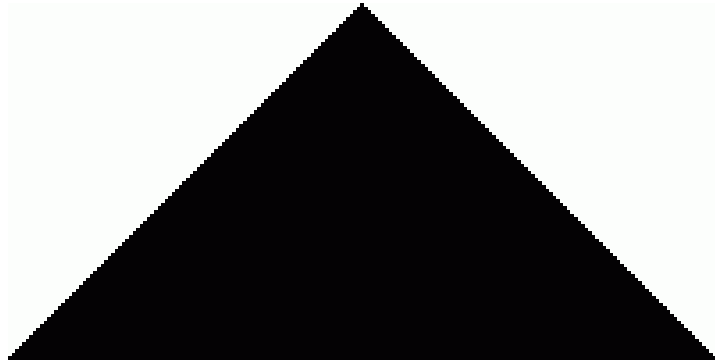


Classes of Behavior

- 4 Main Classes of Behavior Can be identified
- Class 1:
 - Global uniform patterns
 - E.g., all cells “white”
- Class 2:
 - Periodic, regular patterns
 - E.g., regular repetitive triangles
- Class 3:
 - Seemingly random
 - We now it is not random, but it appears like it is, no recognizable patterns
- Class 4:
 - Complex, self-organized behavior
 - Organized, recognizable structured in the middle of chaos
 - Though never really repetitive

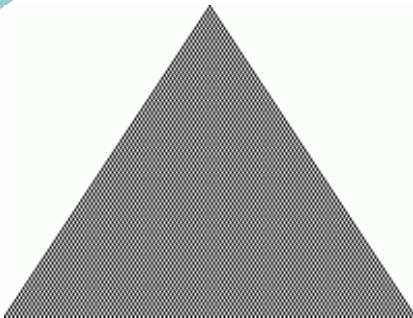
Class 1 Example

Rule 254

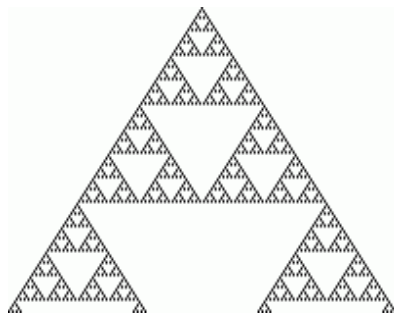


Class 2 Examples

Rule 250

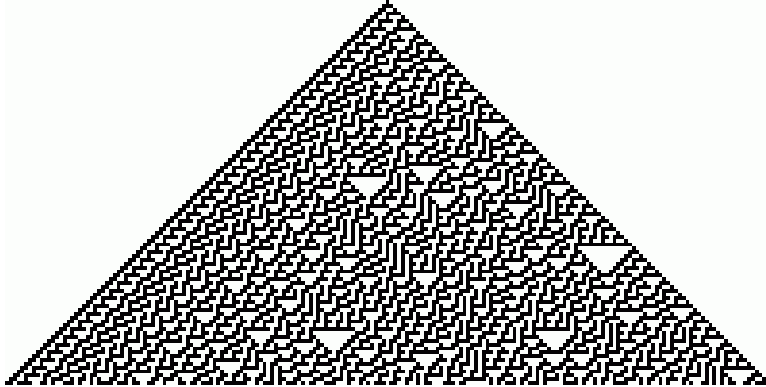


Rule 90



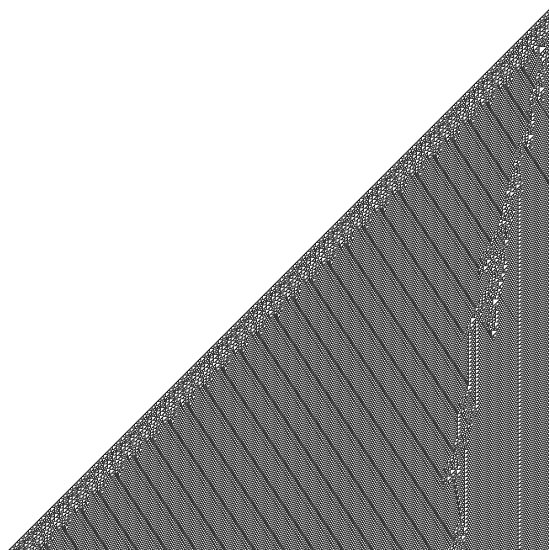
Class 3 Example

Rule 30



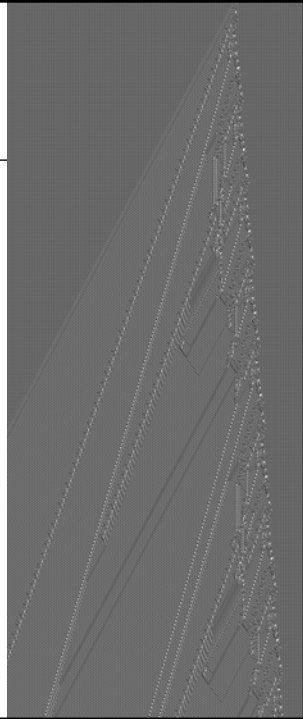
Class 4 Example (1)

Rule 110

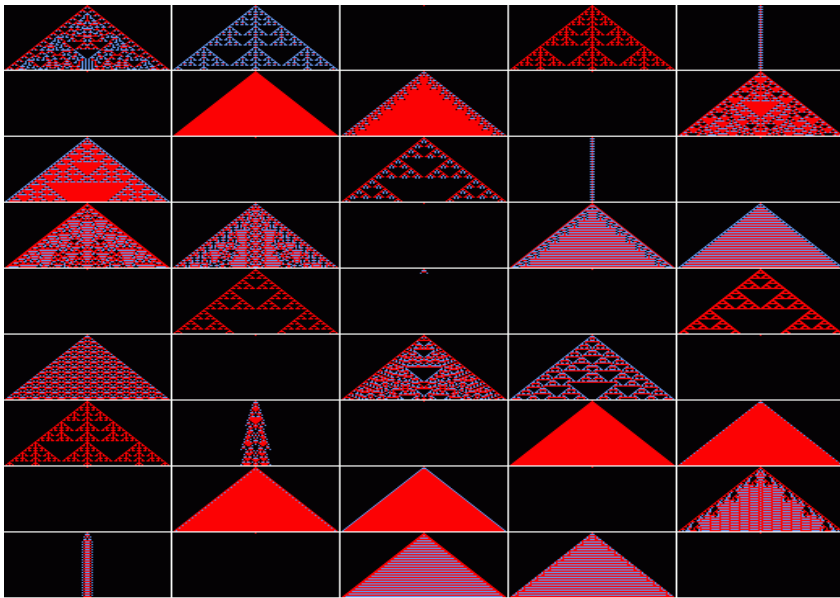


Class 4 Example (2)

Structures in Rule 110



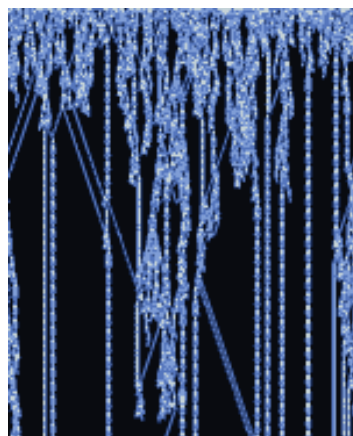
The 4 Classes in 3-color Totalistic CA



The 4 Classes in Multicolor Totalistic CA



Structures in Class 4 CA





Formalizing CA Classes

- What determines the class to which a CA belongs?
- In general, in most complex systems
 - A limited set of parameter (and often a single parameter) can be identified
 - That determines the type of dynamic behavior
- This parameter is a sort of “measure of complexity of the system”
 - And is typically denoted with “lambda” λ



The Lambda of CA

- Let us consider a simple 1-D CA with binary state
 - λ is the probability that a cell will be in state “1” in the next iteration
- It is a sort of measure of the complexity of the system
 - It measure how strong is the degree of interaction between cells
 - It measure the amount of “feedback” in the systems

CA Classes and λ

- λ close to zero
 - All cells die
 - Not enough feedback to keep the system "alive"
 - **Class 1**
- λ higher than zero and less than 0,3
 - Periodic patterns
 - Enough feedback to keep the system alive
 - Still limited enough to avoid complex interactions
 - **Class 2**
- λ closer to 0,3
 - Stable not-periodic structures emerge
 - The interaction feedback is quite strong, but it still preserve the possibility for regular self-organized structures to exists
 - **Class 3**
- λ high than 0,3
 - Random, chaotic behavior
 - To much feedback "noise" to sustain any regular behavior
 - **Class 4**

CA Classes in the Elementary CA

- Rule 32: $\lambda=0,03$



- Rule 90: $\lambda=0,2$



- Rule 110: $\lambda=0,3$

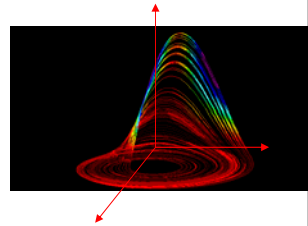
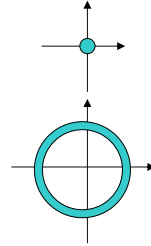


- Rule 30: $\lambda=0,5$



CA and Attractors

- Reasoning with reference to some sort of "phase space" for CA
 - Class 0: a single limit point attractor
 - The system stabilize in a static way
 - Class 1: one or several limit cycles
 - The system stabilize in a periodic dynamic pattern
 - Class 3: complex attractors
 - The systems evolves in an attractor which is a complex manifold, but which still ensures some sort of "bounded" behavior
 - Class 6: chaotic attractor
 - The system evolves in a very chaotic manifold, with not regularity

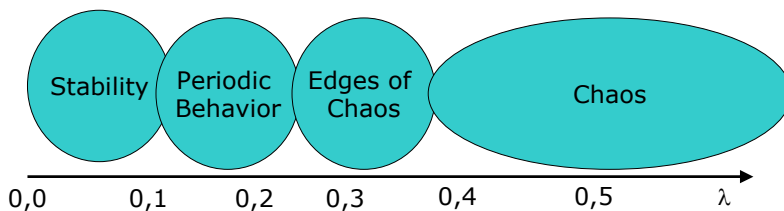


Universality of CA

- All types of CA, defined a proper λ , exhibit very similar behavior
 - A complex CA does not lead to more complex behavior
 - A behavior is always in one of the 4 classes
- All types of complex systems exhibit similar characteristics
 - Some sort of λ exists such that
 - Its tuning move the system from simple, to periodic, to complex, to chaotic behavior

The Edges of Chaos

- For any type of complex system, we can identify “regions” of behavior in function of some relevant parameters (e.g., λ)
 - The shift from order to chaos involves a region in which complex behavior emerges
 - The Edge of Chaos



Adaptive Self-organization and the Edges of Chaos

- At “the edges of chaos” a system exhibit complex patterns
 - That although never repetitive
 - Shows some regularity
 - Have spontaneously emerged
 - There is some sort of “self-organization”
- The patterns live and survive in a continuously perturbed world
 - Neighbor cells continuously interact with the pattern
 - And the pattern nevertheless survive, possibly changing its shape
 - ADAPTIVITY!!!
- Actually, most interesting natural systems are at the “edges of chaos”
 - The heart
 - Living organisms
 - The Internet and Complex Networks
 - Will see in the following of the course...



Part 3

- Asynchronous and Stochastic CA



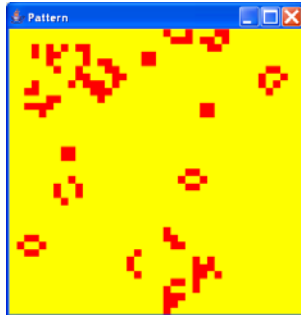
What are Asynchronous CA

- Asynchronous CA, unlike Synchronous one
 - Are not globally synchronized for state transitions
- Each cell evaluate the state transition function independently of other cells
 - And then change its state immediately
- When a cell change its state?
 - When commanded by an external entity
 - Not really an interesting case
 - Autonomously, based on some internal autonomous decision making
 - Randomly from time to time
 - Based on an internal clock
- Asynchronous CA provides for a more continuous notion of time
 - The time is no longer a sequence of time frame
 - But it is a **continuum**, in which cells acts independently of each other

Dynamics of Asynchronous CA (1)

- The Behavior of Synchronous CA is dramatically different from that of Asynch
- Consider, e.g., the “Game of Life” rules

Synchronous Evolution



Asynchronous Evolution

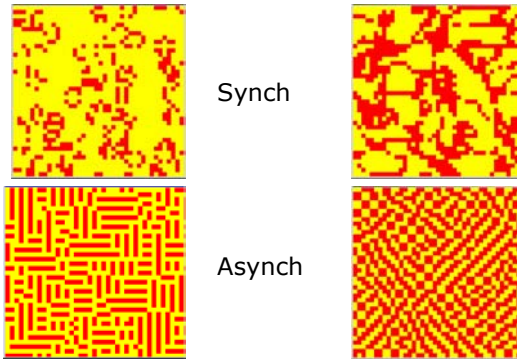


Dynamics of Asynchronous CA (2)

- In general
 - A final state that is stable for a synchronous CA is stable also for an asynchronous CA
- The attractors are the same
- However
 - The evolution path of an asynchronous CA is much different
 - Tends to reach different final configurations
- First, it has a natural endeavor to self-organize itself
 - Some sorts of “embryonic” patterns emerge
 - For any rule
- And tends to be “stable”
 - The evolution stabilize very rapidly
 - While synchronous CA tend to fall into cyclic attractors

Dynamics of Asynchronous CA (3)

- Other examples of differences between synchronous CA and their asynchronous counterparts



Asynchronous CA and Complexity

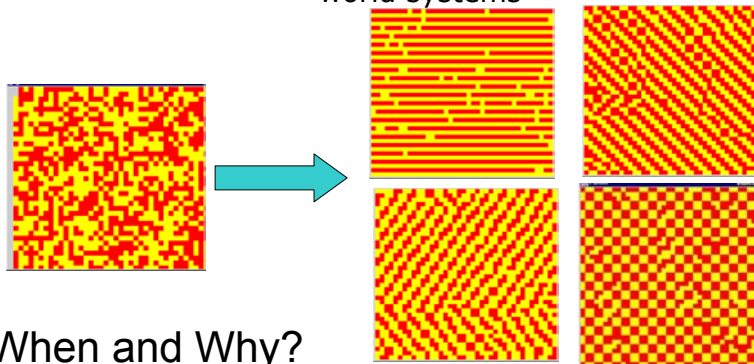
- Asynchronous CA somewhat appears to be more realistic
 - Ecological systems are asynchronous (animals act based on internal clocks)
 - Social systems are asynchronous
 - Distributed systems are asynchronous
- At the same time, they exhibit
- more stable behavior than their synchronous counterparts
- A natural facility in exhibiting simple self organizing patterns
- But where does complexity and adaptivity comes from in the real world ????

Stochastic Cellular Automata (I prefer to call them “Perturbed CA”)

- Consider a normal Cellular automata
 $A=(S,d,V,f)$
- But asynchronous with
 - Cell are autonomous
 - time continuous dynamic
- And in which some stochastic phenomena overrules normal rules for state transitions
 - As if cells were “not rational” OR
 - As if cells were situated in an environment in which external **perturbation** can change arbitrarily the state of cells

Emergent Behaviors in Perturbed CA

- Global scale self-organized behaviors emerge from even simple CA, similar to the ones observed in real world systems



When and Why?

Internal vs. External Dynamics in Perturbed CA

- Let us consider the relative dynamics of
 - The states changing their internal state
 - Probability frequency λ_i
 - The environment forcing some change of state in a cell
 - Probability frequency λ_e
- And evaluate what happens depending on the λ_i/λ_e ratio
 - Which simply characterize the "strength" of the perturbation

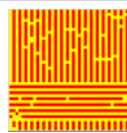
Impact of Perturbation Dynamics



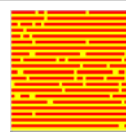
$\lambda_i/\lambda_e = 0.001$



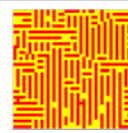
$\lambda_i/\lambda_e = 0.01$



$\lambda_i/\lambda_e = 0.02$

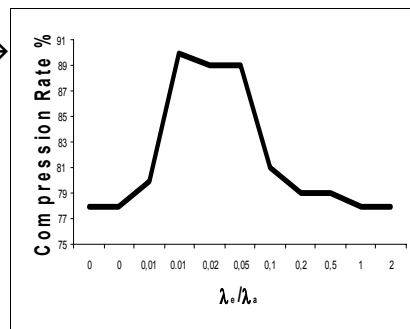


$\lambda_i/\lambda_e = 0.05$



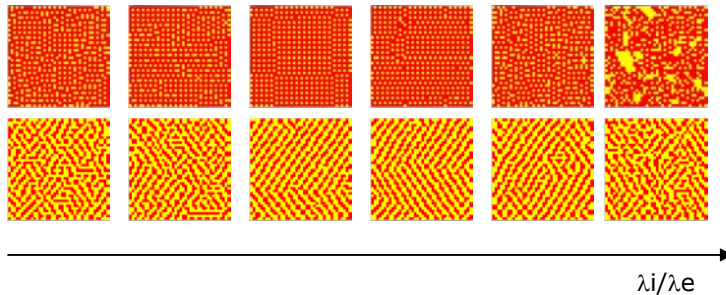
$\lambda_i/\lambda_e = 0.1$

- × Low perturbation dynamics
 - × No global structures emerge → quasi equilibrium
- × Moderate perturbation dynamics
 - × Global structures → spatial patterns
- × High perturbation dynamics
 - Turbulence! → Chaos!



Generality of the Phenomena

- The same phenomena occur for a large variety (nearly all) of asynchronous CA types
- And the same occurs in 1D CA



Implications of the Phenomena (1)

- Real-world system tends to “locally organize”
 - And to stabilize into local self-organizing patterns
 - These may be somewhat “complex” but have no global impact
 - Local minima of equilibrium
- In the presence of environmental dynamics
 - Perturbations always move local configurations out of equilibrium
- Then they tend to re-stabilize but
 - During evolution, only more and more global equilibrium situations survive
 - Until self-organization patterns becomes global
 - In a sort of continuous “simulated annealing”
- Overall, emergence of global spatial patterns from
 - Local interactions
 - External perturbations

Implications of the Phenomena (2)

- The emergence of global spatial patterns in perturbed CA is the phenomena that occur in:
 - Sand dunes, granular media, population distribution, etc.
- So, we have eventually completed the pictures on CA as complex systems,
 - Showing the impact of the last ingredient of the “complexity” recipe (openness and environmental dynamics)



Part 4

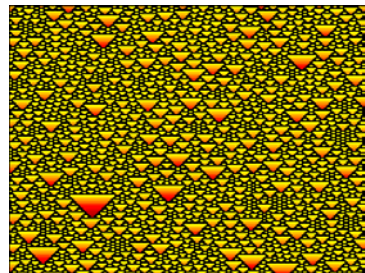
- Applications of CA

What are CA useful for?

- For the study of complex systems
 - We have already seen in the previous part
 - CA as paradigmatic of more complex complex systems
- For the understanding and simulation of complex natural behavior
 - Chemical, physical, ecological, biological
- For the study of complex distributed systems
 - Multiagent systems
 - Complex networks
 - Information and virus propagation

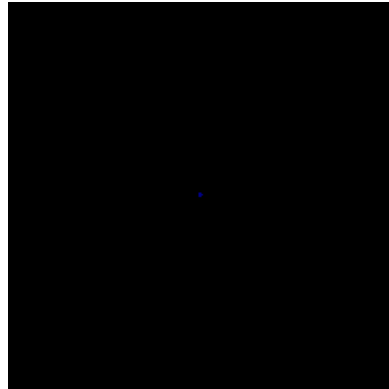
Phenomena of Growth: Cone Shell Patterns

- The shell grows up linearly at its borders
 - Pigmentation at a specific point is influenced by the pigmentation around the point of growth
- It's a 1-D CA?
 - Yes, to some extent
 - Although pigmentation is not really discrete, by subject to chemical diffusion
- Reaction-diffusion model is better
 - Will see in the future



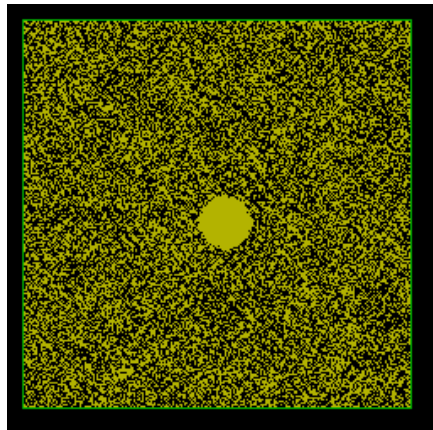
Phenomena of Growth: Snowflakes

- New ice crystals attach to the existing structure
 - With simple "CA-like" rules
 - On a hexagonal lattice



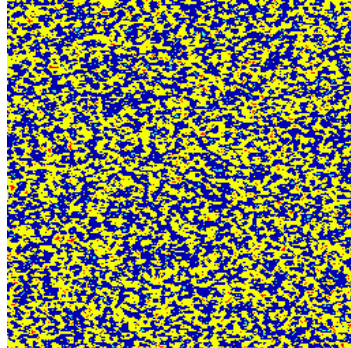
Physical Phenomena: Wave diffusion

- Waves propagates in a simple way
 - Via local interactions among particles
- Can be simulated via a simple CA
 - The state of a local cell representing the pressure



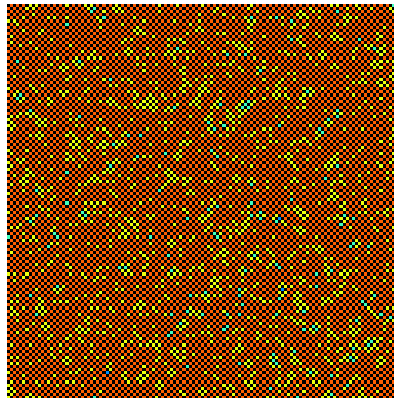
Chemical Phenomena: Simulating a Reaction

- Chemical reactions take place via local interaction of chemical components in a space
- Can be simulated via 2-D CA
 - The state of a CA represent the concentration of components in a point
 - Transition rules determines how concentrations changes during the reactions



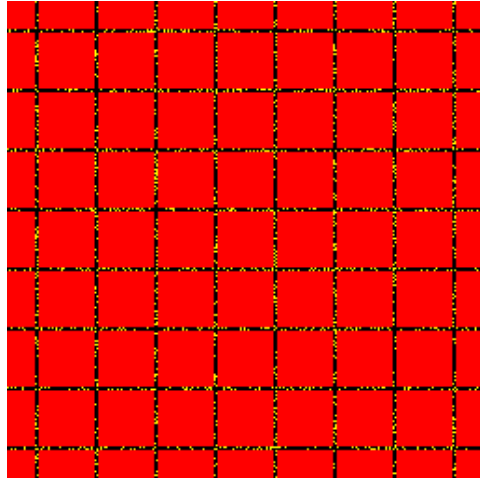
Ecological Simulation: Forest in Fire

- The state of the cell determines the presence of tree and of fire in the forest
- Rules specify how fire propagates



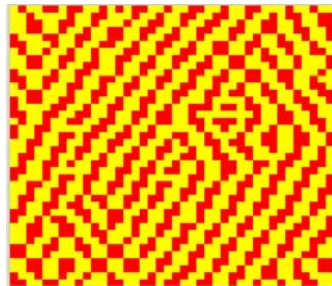
Social Phenomena: Traffic Simulation


- Traffic Simulation
 - The state of cells determines the presence of vehicles
- Rule transitions determines how a vehicle can move
 - i.e., how a state can be transferred to a close cells



Perturbed CA and Patterning of Animals


- Strip like Patterning of Animals
 - Cells grows and pigments diffuse in a continuously perturbed environment
 - The results is a sort of globally organized pattern





Perturbed Asynchronous CA and Modern Distributed Systems (1)

- Asynchronous CA resembles network of agents
 - Autonomous components (their actions, i.e., state transitions, are not subject to a global flow of control)
 - Interacting with each other in a local way
 - As multiagent systems and, more generally, modern distributed & decentralized computing systems
- And they are situated too
 - Their actions consider and are influenced by the environment in which they situate
 - Influencing state transitions
- So, asynchronous perturbed CA are a sort of “minimalist” modern distributed system



Perturbed Asynchronous CA and Modern Distributed Systems (2)

- Given the similarities
- It is likely that the same types of globally self-organized behaviors emerge in large open distributed systems
 - Global patterns of coordination of activities
 - Global patterns of information flow
- This can be **dangerous**
 - Unpredictable “resonance” like behavior, depending on the environmental dynamics
 - E.g., global price imbalances in computational markets, global imbalances in resource exploitations, biased sensing activities in sensor networks
- But it can be also **advantageous**
 - E.g., global (indirect) coordination achieved by “injecting” energy via the the environment
 - A very simple and effective way to achieve global scale coordination and/or global scale diffusion of information



Conclusions and Open Issues

- CA are a very important class of complex systems
 - Representative of a larger class of systems
 - Useful for simulation
 - With possible implications for modern distributed systems
- But there are issues that CA are not able to consider
 - CA use only regular grid-like networks
 - What is the influence of the network structure?
 - What is the influence of non-local network connections?
 - How do networks of interactions actually arise?
 - CA does not consider other means of interactions
 - E.g. diffusion of properties across the environment, as in ants and field-based systems
 - CA does not consider dynamic networks
 - E.g., mobility of cells or of components across the networks, dynamic changes in the network structure